YEAR 12 MATHEMATICS: APPLICATIONS
Test 2 (Sequences) Semester 1 2018

SHENTON COLLEGE

NAME: Solutions

TEACHER: McRae Mackenzie Ryan Staffe

No Calculator No notes Formula sheet provided Total time: 20 minutes

TOTAL $\overline{54}$

$\overline{21}$

QUESTION 1 [10 marks - 2, 2, 2, 2, 2]

State whether the following are arithmetic or geometric progressions or neither and give a reason for your choice.

✓ states progression
✓ reason (must be a clear reason showing reference to successive terms and value change)

a) 19, 16, 13, 10, ...

A.P. ✓

common diff. of −3 ✓

b) 8, 12, 18, 27, ...

G.P. ✓

common ratio of 1.5 ✓

c) $\frac{1}{12}, \frac{1}{6}, \frac{1}{4}, \frac{1}{3}$ ....

A.P. ✓

common diff. of $\frac{1}{12}$ ✓

d) $5x$, $5x^2$, $5x^4$, $5x^8$, ...

Neither ✓

no common diff. or ratio ✓

e) 0.06, 0.18, 0.54, 1.62, ...

G.P. ✓

common ratio of 3 ✓

QUESTION 2 [4 marks - 2, 2 marks]

Samantha starts working on 1st January 2002 and is paid in the first year a yearly salary of $32,000 with an annual increase of $1800 per year.

a) Write down a general rule for determining Samantha's salary at T_n where n is the n^{th} year of Samantha's service.

$T_n = 32000 + (n-1)1800$

✓ correct rule

✓ correct sub.

b) What salary will Samantha receive in her tenth year of service?

$T_{10} = 3200 + (10-1)1800$

$= \$48\,200$

✓ correct sub.

✓ answer

QUESTION 3 [2 marks]

The following three numbers are chosen, where N stands for a number :

4 , N , 8

If **any order** of the three numbers is possible, write down all the values of N that make the three numbers form an arithmetic sequence.

8 4 N $\therefore N = 0$ | 4 8 N $\therefore N = 12$

8 N 4 $\therefore N = 6$ | 4 N 8 $\therefore N = 6$

N 8 4 $\therefore N = 12$ | N 4 8 $\therefore N = 0$

✓✓ all correct

✓ at least two correct

QUESTION 4 [5 marks - 2, 3]

The sixth term of an arithmetic sequence is 44 and the thirteenth term is -12.

a) Determine the common difference.

$d = \dfrac{-12-44}{13-6}$

$\therefore d = -8$

OR

$44 + 7d = -12$

$7d = -56$

$d = -8$

✓ correct process

✓ answer

b) Write a general rule in terms of n for the sequence.

$a = 44 - 5(-8)$

$a = 84$

$\therefore T_n = 84 + (n-1)(-8)$

or $T_n = -8n + 92$

✓ finds 'a'

✓ correct rule

✓ correct sub.

End of Section 1

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33

Calculator Assumed 1-page A4 single-sided notes Formula sheet provided Total time: 30 minutes

QUESTION 1 [8 marks – 1, 2, 2, 1, 2]

a) For the following sequence, find T_{14}

8, 15, 22, 29, 36,...........

$T_{14} = 99$ ✓

b) Given that $T_n = T_{n-2} + \frac{1}{2}T_{n-1}$ and $T_1 = 2$ and $T_2 = 4$, find the first five terms.

2, 4, 4, 6, 7 ✓ correct terms

(2, 4, 5, 7, 9.5) ← only 1 mark if these answers T_{n-2} and T_{n-1} in wrong order

c) A geometric progression has a third term of 0.5 and a sixth term of 0.0625.

Find i) the common ratio

$0.5 \times r^3 = 0.0625$ ✓ uses algebra

$\therefore r = 0.5$ ✓ solves for 'r'

ii) the seventh term

$T_7 = 0.03125$ ✓

iii) write an explicit rule in terms of n.

$T_n = 2 \times (0.5)^{n-1}$ ✓ Correct 'a'

✓ correct rule.

QUESTION 2 [7 marks - 3, 2, 1, 1]

John buys a new car X for $30,000 which loses 20% of its value per year. At the same time Julie buys a new car Y for $20,000 which loses approximately $1500 of its value per year.

a) Write the **two** recursive rules that represent John and Julie's car values, where n is the number of years since they bought the car.

John: $T_{n+1} = 0.8T_n \quad , \quad T_0 = 30\,000$ ✓✓ correct rules

Julie: $T_{n+1} = T_n - 1500 \quad , \quad T_0 = 20\,000$ ✓ T_0 correct

b) Determine the value of John's car after five years to the nearest $100.

$T_5 = 9830.4$ ✓ value

$\therefore$ $9800 ✓ nearest $100

c) During which year does Julie's car exceed John's in value?

In the 4th year ✓

* must use correct language.

d) John will sell his car during the year when his car is only worth a quarter of its original value. When does this happen?

In the 7th year ✓

QUESTION 3 [9 marks - 1, 1, 2, 2, 2, 1]

Bob and Roberta both colour their hair. The colours they use are not permanent and wash out gradually when they wash their hair. They have 100% colour when they first colour their hair.

(a) The product that Bob uses is such that the colour reduces by 15% of the current amount every time he washes his hair. What percentage of the original level of colour remains after

(i) 1 wash?

85% ✓

(ii) 2 washes?

72.25% ✓

(b) Write an explicit rule for ***n*** washes.

$T_n = 100 \times 0.85^n$ ✓ correct rule

or $T_n = 85 \times 0.85^{n-1}$ ✓ correct "a" and "r"

(c) Bob wants to re-colour his hair **before** the percentage of colour falls below 10%. How many times can he wash his hair before he must colour it again?

$100 \times 0.85^n = 10$ ✓ correct n

$n = 14.17$

$\therefore$ 14 washes ✓ correct washes

(d) Roberta pays more for her hair colouring product and the colour reduces by only 10% of the current amount every time she washes her hair.

(i) What percent remains after ***n*** washes?

$T_n = 100 \times 0.9^n$ ✓ correct 'a' and 'r'

or $T_n = 90 \times 0.9^{n-1}$ ✓ correct rule

(ii) How often does she need to recolour her hair if, like Bob, she does not want the percentage of colour to fall below 10%.

$100 \times 0.9^n = 10$

$n = 21.9$

$\therefore$ 21 washes ✓ correct washes

QUESTION 4 [9 marks – 3, 1, 2, 3]

Derek owns a pool and is always struggling to keep his chlorine levels in the effective range so that it is safe to swim in the pool. A bag of chlorine brings the chlorine levels up by 50 ppm (parts per million) in the pool whenever it is added. Derek must not allow chlorine levels to drop below 70ppm, otherwise algae begins to form in the pool and it is unsafe to swim in. Chlorine levels decrease by 40% each month so Derek decides to add a bag of chlorine at the end of every month.

a) Write a recursive rule that describes this situation if the pool initially starts the month with a level of 140ppm.

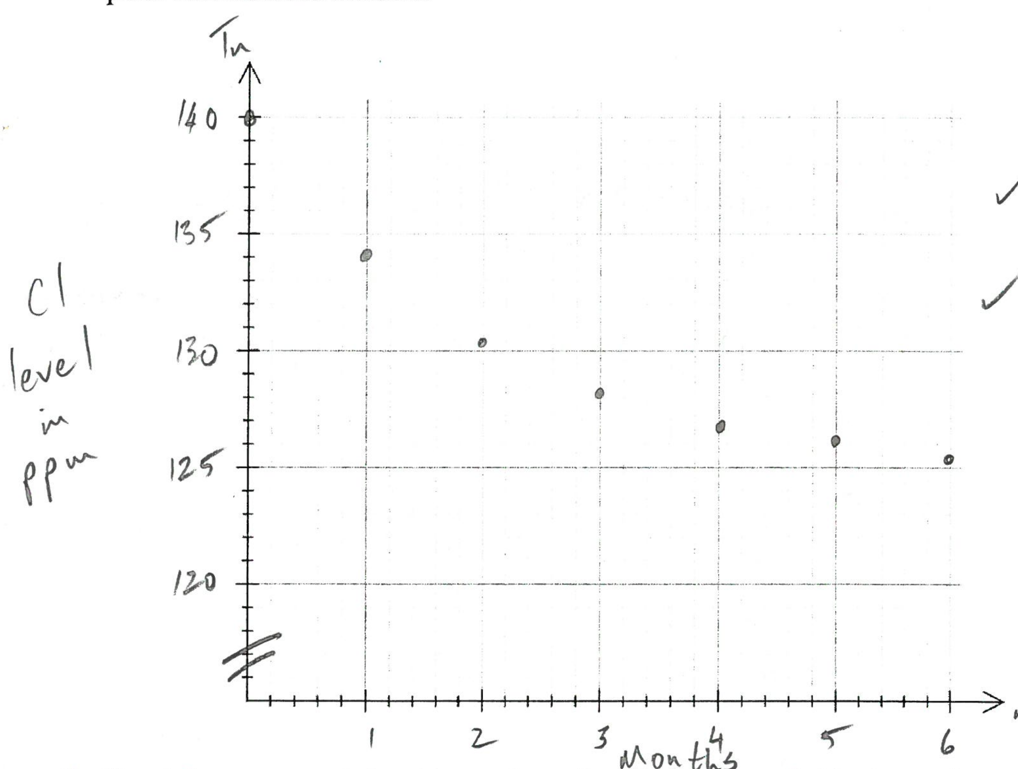
$T_{n+1} = 0.6T_n + 50$ ✓ ratio ✓ constant

$T_0 = 140$ ✓ 1st term T_0

b) How much chlorine will be in the pool after 6 months (to 2 d.p.)?

125.70 ppm ✓

c) Draw a graphical representation of the above situation for the first 6 months of Derek's pool chlorination routine.



✓ label + scale

✓ accuracy

d) Derek is concerned that his routine of adding a bag of chlorine at the end of each month will not keep his pool safe and free of algae in the long term. Is he on the right track with his chlorination routine? Justify your choice with mathematical reasoning.

let $T_{n+1} = T_n = x$ ✓ yes statement

$x = 0.6x + 50$ ✓ steady state solution

$\therefore x = 125$ ✓ finds lowest level of 125ppm

$\therefore$ A steady state solution of 125ppm

Yes, on the right track.

End of Section 2